MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 3, 2016/2017

ETM 4096 - DIGITAL SIGNAL PROCESSING

(All sections / Groups)

30 MAY 2017
9:00 A.M- 11:00 A.M.
(2 Hours)

INSTRUCTION TO STUDENT

1. This question paper consists of 12 pages (including cover page) with 5 questions and 6-page list of formulas given in the appendix.

2. Attempt **ALL** questions. All questions carry equal marks and the distribution of the marks for each question is given.

3. Please write all your answers in the answer Booklet provided.

QUESTION 1

(a) With a block diagram, briefly explain the process of discrete-time digital processing of analog signals.

[4 Marks]

(b) Assume that the following analog signal $x_c(t)$ is sampled at 4 kHz to generate the discrete-time sequence $x[n]$. Determine the expression of $x[n]$.

$$x_c(t) = 3\cos(400\pi t) + 5\sin(1200\pi t) + 6\cos(4400\pi t) + 2\sin(5200\pi t)$$

Does the "aliasing" effect exist in the signal $x[n]$? Justify your answer.

[4 Marks]

(c) Find the impulse response for the causal linear time-invariant (LTI) system satisfying the following difference equation:

$$y[n] - \frac{5}{6}y[n-1] + \frac{1}{6}y[n-2] = \frac{1}{3}x[n-1]$$

[6 Marks]

(d) Consider the system in Fig. Q1.1

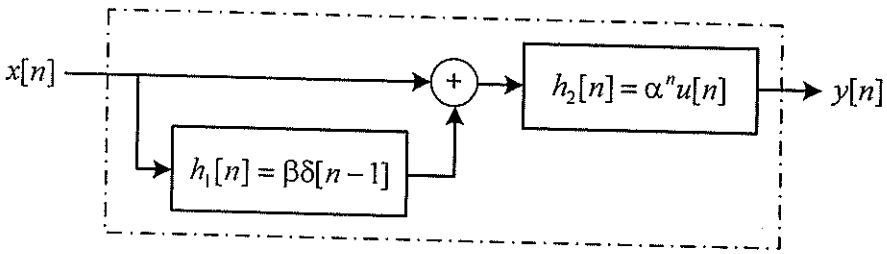


Fig. Q1.1

(i) Find the impulse response $h[n]$ of the overall system.
(ii) Is the system causal? Justify your answer.

[6 Marks]

Continued.....

QUESTION 2

(a) Find the inverse z-transform of

$$X(z)=\frac{-z(z+0.4)}{(z-0.8)(z-2)}$$

if the region of convergence (ROC) is

(i) $|z|>2$
(ii) $|z|<0.8$
(iii) $0.8<|z|<2$

[8 Marks]

(b) Consider the system

$$H(z)=\frac{z\left(z-\frac{1}{2}\right)}{\left(z^3-\frac{27}{8}\right)}$$

(i) Identify the pole(s) and zero(s) and specify all possible ROCs.
(ii) Repeat part (i) for $H^{-1}(z)$.

[6 Marks]

(c) Let $x[n]=(-1)^n u[n-n_0]+\alpha^n u[-n]$. Determine the constraints on the complex number α and the integer n_0 so that the z-transform $X(z)$ exists with the ROC $1<|z|<2$.

[6 Marks]

Continued .....

QUESTION 3

(a) Discuss briefly the conditions necessary for a realizable digital filter to have a linear phase characteristic and the advantages of filters with such characteristic. Classify linear phase filters, and for each type, draw an example of impulse response.

[7 Marks]

(b) A signal is sampled at a frequency of 1 kHz, has a useful content from 0 to 100 Hz and is corrupted with noise from 400 to 500 Hz. Design a digital FIR low-pass filter based on a fixed window to attenuate the noise by at least 53 dB without affecting the useful content by more than 3 dB.

[8 Marks]

(c) Given a desired frequency response $H_d(e^{j\omega})$, show that the rectangular window design minimises the least squares error

$$\varepsilon_{LS} = \frac{1}{2\pi}\int_{-\pi}^{\pi} | H_d(e^{j\omega}) - H_d(e^{j\omega}) |^2 d\omega$$

[5 Marks]

Continued .....

QUESTION 4

(a) A lowpass digital filter is required to have specification shown in Figure Q4, where

$$\delta_p = 0.10875$$
$$\delta_s = 0.17783$$
$$\omega_p = 0.2\pi$$
$$\omega_s = 0.3\pi$$

Design the digital filter using the bilinear z-transformation method on a Butterworth analog filter.

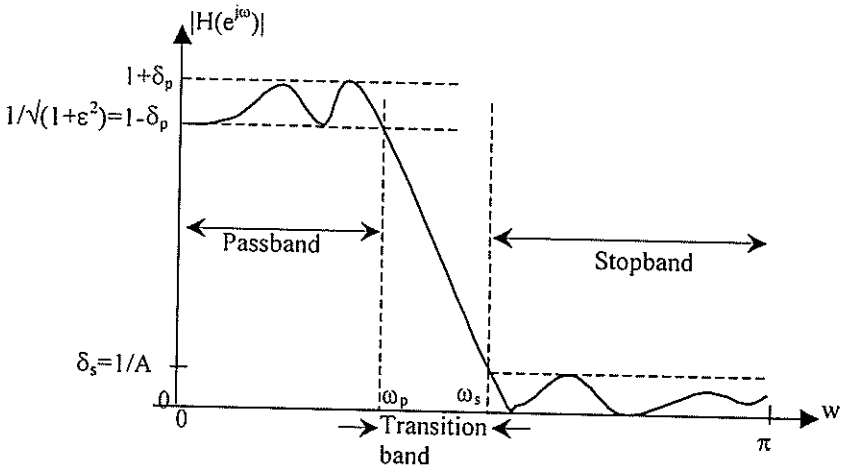


Figure Q4

[12 Marks]

(b) An infinite-impulse-response (IIR) system can be represented by several equivalent flow-graph network structures. There are basically three forms of representing IIR systems. Draw and label the structures for the following cases.

(i) $H(z) = \dfrac{\sum_{k=0}^{p} b_k z^{-k}}{\sum_{k=0}^{q} a_k z^{-k}}$, in Direct Form 1

(ii) $H(z) = \left(\dfrac{b_{01} + b_{11}z^{-1} + b_{21}z^{-2}}{1 - a_{11}z^{-1} - a_{21}z^{-2}}\right)\left(\dfrac{b_{02} + b_{12}z^{-1} + b_{22}z^{-2}}{1 - a_{12}z^{-1} - a_{22}z^{-2}}\right)\left(\dfrac{b_{03} + b_{13}z^{-1} + b_{23}z^{-2}}{1 - a_{13}z^{-1} - a_{23}z^{-2}}\right)$, in cascade form

(iii) $H(z) = \gamma_0 + \dfrac{\gamma_{01}}{1 + \alpha_{11}z^{-1}} + \dfrac{\gamma_{02} + \gamma_{12}z^{-1}}{1 + \alpha_{12}z^{-1} + \alpha_{22}z^{-2}}$, in parallel form

[8 Marks]

Continued .....

QUESTION 5

(a) Determine the eight-point DFT of the signal $x[n] = \{1, 1, 1, 1, 1, 1, 0, 0\}$.

[8 Marks]

(b) Two finite-length sequences, $x_1[n]$ and $x_2[n]$, that are zero outside the interval [0, 99] are circularly convolved to form a new sequence $y[n]$.

$$y[n] = x_1[n] \textcircled{N} x_2[n]$$

where $N = 100$. If $x_1[n]$ is nonzero only for $10 \leq n \leq 39$, determine the values of n for which $y[n]$ is guaranteed to be equal to the linear convolution of $x_1[n]$ and $x_2[n]$.

[4 Marks]

(c) Derive an 8-point fast Fourier transform (FFT) and draw the butterfly diagram. Use decimation in frequency.

[8 Marks]

Continued...

APPENDIX

SUMMARY OF USEFUL TABLES

TABLE A1. DISCRETE-TIME FOURIER TRANSFORM PAIRS

Sequence	Discrete-Time Fourier Transform
$\delta[n]$	1
$\delta[n-n_o]$	$e^{-j\omega n_o}$
$1 \quad (-\infty < n < \infty)$	$\sum_{k=-\infty}^{\infty} 2\pi\delta(\omega + 2\pi k)$
$a^n u[n] \quad (\lvert a\rvert < 1)$	$\frac{1}{1-ae^{-j\omega}}$
$u[n]$	$\frac{1}{1-e^{-j\omega}} + \sum_{k=-\infty}^{\infty} \pi\delta(\omega + 2\pi k)$
$(n+1)a^n u[n] \quad (\lvert a\rvert < 1)$	$\frac{1}{(1-ae^{-j\omega})^2}$
$\frac{r^n \sin\omega_p(n+1)}{\sin\omega_p} u[n] \quad (\lvert r\rvert < 1)$	$\frac{1}{1-2r\cos\omega_p e^{-j\omega} + r^2 e^{-j\omega})}$
$\frac{\sin\omega_c n}{\pi n}$	$X(e^{j\omega}) = \begin{cases} 1, & \lvert\omega\rvert < \omega_c \\ 0, & \omega_c < \lvert\omega\rvert < \pi \end{cases}$
$x[n] = \begin{cases} 1, & 0 \le n \le M \\ 0, & otherwise \end{cases}$	$\frac{\sin[\omega(M+1)/2)]}{\sin(\omega/2)} e^{-j\omega M/2}$
$e^{j\omega_0 n}$	$\sum_{n=-\infty}^{\omega} 2\pi\delta(\omega - \omega_0 + 2\pi k)$
$\cos(\omega_0 n + \phi)$	$\sum_{n=-\infty}^{\infty} [\pi e^{j\phi}\delta(\omega - \omega_0 + 2\pi k) + \pi e^{-j\phi}\delta(\omega + \omega_0 + 2\pi k)]$

TABLE A2. SOME COMMON z-TRANSFORM PAIRS

The z-transform

Properties of the z-transform

Property	$x[n]$	$X(z)$	$\mathcal{R}_x$
Linearity	$ax[n]+by[n]$	$aX(z)+bY(z)$	$\mathcal{R}_x \cap \mathcal{R}_y$
Time shifting	$x[n-m]$	$z^{-m}X(z)$	$\mathcal{R}_x$
Time reversal	$x[-n]$	$X(z^{-1})$	$1/\mathcal{R}_x$
Convolution	$x[n]*y[n]$	$X(z)Y(z)$	$\mathcal{R}_x \cap \mathcal{R}_y$

Common z-transform pairs

$x[n]$	$X(z)$	$\mathcal{R}_x$
$\delta[n]$	1	$\forall z$
$\delta[n-n_0]$	z^{-n_0}	Possibly $\forall z$
$u[n]$	$\frac{1}{1-z^{-1}}$	$\lvert z\rvert > 1$
$-u[-n-1]$	$\frac{1}{1-z^{-1}}$	$\lvert z\rvert < 1$
$a^n u[n]$	$\frac{1}{1-az^{-1}}$	$\lvert z\rvert > \lvert a\rvert$
$-a^n u[-n-1]$	$\frac{1}{1-az^{-1}}$	$\lvert z\rvert < \lvert a\rvert$
$na^n u[n]$	$\frac{az^{-1}}{(1-az^{-1})^2}$	$\lvert z\rvert > \lvert a\rvert$
$-na^n u[-n-1]$	$\frac{az^{-1}}{(1-az^{-1})^2}$	$\lvert z\rvert < \lvert a\rvert$

TABLE A3. SOME COMMON WINDOWS FOR FIR FILTER DESIGN

Ideal bandpass, n=0, 1, 2 …., N (Filter order given by N)

$$h[n]=\frac{w_2}{\pi}\sin c\left(\frac{w_2\left(n-\frac{N}{2}\right)}{\pi}\right)-\frac{w_1}{\pi}\sin c\left(\frac{w_1\left(n-\frac{N}{2}\right)}{\pi}\right)$$

Rectangular	$w[n]=\begin{cases}1 & 0\le n\le N\\ 0 & else\end{cases}$
Hanning	$w[n]=\begin{cases}0.5-0.5\cos\left(\frac{2n\pi}{N}\right) & 0\le n\le N\\ 0 & else\end{cases}$
Hamming	$w[n]=\begin{cases}0.54-0.46\cos\left(\frac{2n\pi}{N}\right) & 0\le n\le N\\ 0 & else\end{cases}$
Blackman	$w[n]=\begin{cases}0.42-0.5\cos\left(\frac{2n\pi}{N}\right)+0.08\cos\left(\frac{4n\pi}{N}\right) & 0\le n\le N\\ 0 & else\end{cases}$
Kaiser	$w[n]=\frac{I_0\left(\beta\sqrt{1-\left(\frac{n-\alpha}{\alpha}\right)^2}\right)}{I_0(\beta)}$ $\quad 0\le n\le N \quad \alpha=N/2$ where $\beta=\begin{cases}0.1102(A-8.7), & A>50\\ 0.5842(A-21)^{0.4}+0.07886(A-21), & 21\le A\le 50\\ 0, & A<21\end{cases}$ $N=\begin{cases}(A-7.95)/(2.285\Delta\omega), & A\ge 21\\ 5.655/\Delta\omega, & A<21\end{cases}$

TABLE A4. COMPARISON OF COMMONLY USED WINDOWS

Type of Window (Filter length = N+1)	Peak Side-Lobe Amplitude (Relative)	Approximate Width of Main Lobe	Stopband Attenuation, $20\log_{10}\delta$ (dB)	Transition Width ($\Delta\omega$ $\lvert w_p - w_s \rvert$)
Rectangular	-13	$4\pi/(N+1)$	-21	$1.8\pi/N$
Bartlett	-25	$8\pi/N$	-25	
Hanning	-31	$8\pi/N$	-44	$6.2\pi/N$
Hamming	-41	$8\pi/N$	-53	$6.6\pi/N$
Blackman	-57	$12\pi/N$	-74	$11\pi/N$

TABLE A5. IIR FILTER DESIGN

TABLE A5.1. THE COEFFICIENTS IN THE SYSTEM FUNCTION OF A NORMALIZED BUTTERWORTH LOW PASS FILTER (Ω_c=1) FOR ORDERS $1 \le N \le 8$

N	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8
1	1.0000							
2	1.4142	1.0000						
3	2.0000	2.0000	1.0000					
4	2.6131	3.4142	2.6131	1.0000				
5	3.2361	5.2361	5.2361	3.2361	1.0000			
6	3.8637	7.4641	9.1416	7.4641	3.8637	1.0000		
7	4.4940	10.0978	14.5918	14.5918	10.0978	4.4940	1.0000	
8	5.1258	13.1371	21.8462	25.6884	21.8462	13.1372	5.1258	1.0000

TABLE A5.2. CLOSED-FORM EXPRESSION FOR SOME USEFUL SERIES

$$\sum_{n=0}^{N-1} a^n = \frac{1-a^N}{1-a}$$

$$\sum_{n=0}^{N-1} na^n = \frac{(N-1)a^{N+1} - Na^N + a}{(1-a)^2}$$

$$\sum_{n=0}^{N-1} n = \frac{1}{2}N(N-1)$$

$$\sum_{n=0}^{\infty} a^n = \frac{1}{1-a} \qquad |a| < 1$$

$$\sum_{n=0}^{\infty} na^n = \frac{a}{(1-a)^2} \qquad |a| < 1$$

$$\sum_{n=0}^{N-1} n^2 = \frac{1}{6}N(N-1)(2N-1)$$

$$\sum_{n=N_1}^{N_2} a^n = \frac{a^{N_1-1} - a^{N_2}}{1-a}$$

TABLE A6. PARAMETERS OF ANALOG FILTER PROTOTYPES

Discrimination factor $d = \left[\dfrac{(1-\delta_p)^{-2}-1}{\delta_s^{-2}-1}\right]^{1/2}$

Selectivity factor $k = \dfrac{\Omega_p}{\Omega_s}$

Butterworth	Order, $N \geq \dfrac{\log d}{\log k}$ System function, $H_a(s) = \prod_{k=0}^{N-1} \dfrac{-s_k}{s - s_k}$ where $s_k = \Omega_c e^{j(2k+1+N)\pi/(2N)}$
Chebyshev-I	Order, $N \geq \dfrac{\cosh^{-1}(1/d)}{\cosh^{-1}(1/k)}$ System function, $H_a(s) = H_a(j0) \prod_{k=0}^{N-1} \dfrac{-s_k}{s - s_k}$ where $s_k = -\Omega_p \sinh[\sinh^{-1}(1/\epsilon)/N] \sin[(2k+1)\pi/(2N)]$ $+ j\Omega_p \cosh[\sinh^{-1}(1/\epsilon)/N] \cos[(2k+1)\pi/(2N)]$ $\epsilon = \sqrt{(1-\delta_p)^{-2}-1}$ $H_a(j0) = \begin{cases} 1, & \text{odd } N \\ (1+\epsilon^2)^{-0.5}, & \text{even } N \end{cases}$
Chebyshev-II	Order, $N \geq \dfrac{\cosh^{-1}(1/d)}{\cosh^{-1}(1/k)}$ System function, $H_a(s) = \prod_{k=0}^{N-1} \dfrac{s_k}{s'_k}\left(\dfrac{s - s'_k}{s - s_k}\right)$ where $s_k = \Omega_s\{-\sinh[\sinh^{-1}(1/\epsilon)/N] \sin[(2k+1)\pi/(2N)]$ $+ j \cosh[\sinh^{-1}(1/\epsilon)/N] \cos[(2k+1)\pi/(2N)]\}^{-1}$ $s'_k = j\Omega_s / \cos((2k+1)\pi/(2N))$ $\epsilon = 1/\sqrt{\delta_s^{-2}-1}$

Elliptic	$N \geq \dfrac{\log(16/d^2)}{\log(1/q)}$, $q = q_o + 2q_o^5 + 15q_o^9 + 150q_o^{13}$, $q_o = \dfrac{1}{2}\left[\dfrac{1-(1-k^2)^{1/4}}{1+(1-k^2)^{1/4}}\right]$

TABLE A7. DISCRETE-TIME FOURIER ANALYSIS

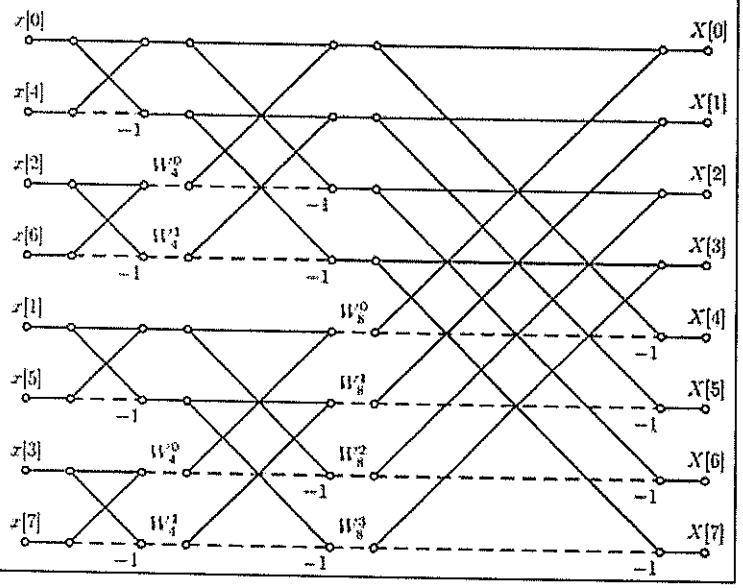
The discrete Fourier transform (DFT)

$$X[k] = \sum_{n=0}^{N-1} x[n]e^{-j2\pi kn/N}, \quad k = 0, 1, \ldots, N-1$$

Properties of the DFT

Property	$x[n]$	$X[k]$
Linearity	$A_1x_1[n] + A_2x_2[n]$	$A_1X_1[k] + A_2X_2[k]$
Time shifting	$x[\langle n - n_0\rangle_N]$	$X[k]W_N^{kn_0}$
Frequency shifting	$x[n]W_N^{-k_0n}$	$X[\langle k - k_0\rangle_N]$
Time reversal	$x[\langle -n\rangle_N]$	$X[\langle -k\rangle_N]$
Conjugation	$x^*[n]$	$X^*[\langle -k\rangle_N]$
Convolution	$x[n] \circledast y[n]$	$X[k]Y[k]$
Modulation	$Nx[n]y[n]$	$X[k] \circledast Y[k]$

The decimation-in-time FFT



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